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Electrostatics and kinetics of two-dimensional electrons in lateral superlattices on vicinal planes

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Abstract. The potential of a one-dimensional lateral superlattice screened by two-dimensional electron gas located in close proximity is found. The periodic potential created by the superlattice affects the kinetic and optical properties of the electron system. The magnetoreflectance, Faraday rotation angle and ellipticity of the reflected electromagnetic field are calculated.

1. Introduction

One of the possible ways to fabricate a short-period lateral superlattice (SL) is via segregation of charged impurities on vicinal planes of crystals (see figure 1). The terraced interface of a heterojunction is populated by donors nonuniformly. The donors tend to aggregate at edges of the terraces forming chains of positive charges. As a result, 2D electrons residing close to the interface 'see' a one-dimensional periodic potential $V(x)$ with period a determined by the angle of misorientation of the vicinal plane (see, e.g., [1–3]). Transport properties of 2D electron gas with a 1D lateral SL have been widely discussed in the literature. The most remarkable effect is the occurrence of Weiss oscillations of magnetoresistivity [4–8]. The

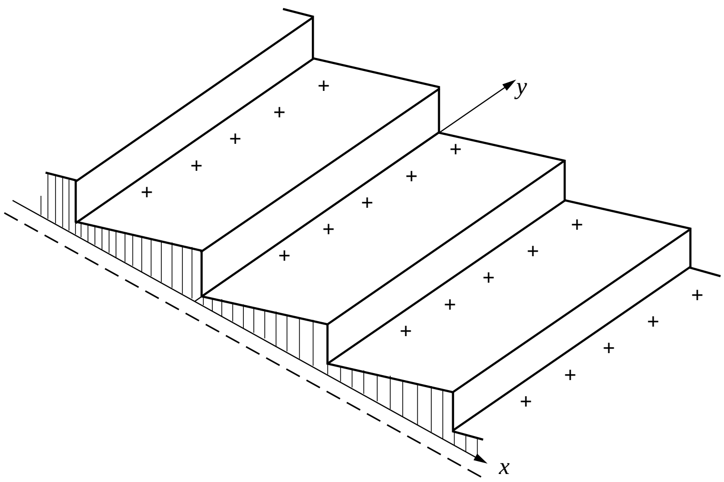

Figure 1. Two-dimensional electron gas at a vicinal surface: +: charged donors; −: electrons.

high-frequency conductivity of a lateral SL has been investigated in [9] (see also [10]). In the theoretical papers cited above, the potential $V(x)$ is represented by its Fourier components $V^{(r)}$ which are just input parameters of the theory. In the present paper, we first calculate the potential $V(x)$, taking into account the screening effects in 2D electron gas. Then we consider magneto-optical phenomena in a 1D lateral SL.

2. The screened potential of a 1D SL

Consider a periodic set of charged filaments parallel to the y-axis and lying in the plane $z = 0$. The linear charge density on each filament is ξ. In the plane $z = -\Delta$, we have a strongly degenerate 2D electron gas. The problem is that of finding the self-consistent potential $V(x)$ and surface density of electrons $\sigma(x)$ in the plane $z = -\Delta$. Of course, we realize that this is an idealized formulation of the problem. In reality there are inevitable charge-density fluctuations, so ξ is not constant and neither are all filaments identical. In fact, we look for a macroscopically averaged potential $V(x)$. The above-mentioned fluctuations contribute to electron scattering.

Note, first of all, that in the classical limit ($\hbar \to 0$) one can neglect the Thomas–Fermi screening radius which, for degenerate 2D electrons, is just $a_0^*/2$, where a_0^* is the effective Bohr radius $a_0^* = \hbar^2\varepsilon/m^*e^2$ and ε is the dielectric permeability. Then the problem formulated above can be solved straightforwardly: the plane $z = -\Delta$ is equipotential, $V(x) = \text{constant} = 0$ and the surface charge density $\sigma(x)$ can be found by the mirror image method. For a single filament placed at $x = na$, $z = 0$ (n is an integer), we get

$$\sigma_n(x) = \frac{\xi}{\pi}\,\frac{\Delta}{(x-na)^2+\Delta^2}. \tag{1}$$

Then the total surface charge density is just $\sum_n \sigma_n$:

$$\sigma(x) = \frac{\xi}{2a}\,\frac{\sinh(2\pi\Delta/a)}{\cosh^2(\pi\Delta/a)-\cos^2(\pi x/a)}. \tag{2}$$

Of course, the superposition principle $\sum_n \sigma_n$ is applicable, because we deal with linear equations of electrostatics. The electrostatic problem with *finite* screening is, generally speaking, nonlinear, since the charge density depends on the potential. However, a lucky exception is the Thomas–Fermi limit for the 2D electron gas. Indeed, the surface density of 2D electrons $n(\boldsymbol{r})$ is given by

$$n(\boldsymbol{r}) = 2\int \frac{\mathrm{d}^2\boldsymbol{p}}{(2\pi\hbar)^2}\; f\left[\frac{p^2}{2m^*} - e\varphi(\boldsymbol{r})\right] \tag{3}$$

where f is the Fermi occupation number, $\varphi(\boldsymbol{r})$ is the electrostatic potential, e is the absolute value of the electron charge and $\boldsymbol{p}$ is the 2D momentum. Thus, for $T \to 0$

$$n(\boldsymbol{r}) = \frac{m^*T}{\pi\hbar^2}\ln\left[1+\exp\left(\frac{\mu+e\varphi}{T}\right)\right] \to \frac{m^*}{\pi\hbar^2}\,[\mu+e\varphi(\boldsymbol{r})]\,\vartheta\,[\mu+e\varphi(\boldsymbol{r})] \tag{4}$$

where $\vartheta(t)$ is the Heaviside step function and μ is the chemical potential. The potential $\varphi(\boldsymbol{r})$ consists of two parts: the induced potential φ_{ind} created by redistribution of 2D electrons and the external potential φ_{ext} resulting from the outer charges. If this part contains no repulsion contributions, the density $n(\boldsymbol{r})$ is nonzero everywhere and, hence, $\mu+e\varphi$ is always positive and ϑ equals 1 for the entire space. Thus, the Thomas–Fermi equation formally becomes linear.

The Green function of this equation for the 2D electron gas is known [11]. The Fourier component of the potential created by a point charge Q placed at a distance Δ above the 2D degenerate electron gas is given by

$$v(k) = \frac{2\pi Q\mathrm{e}^{-k\Delta}}{k+\kappa} \qquad \kappa = \frac{2}{a_0^*}. \tag{5}$$

Correspondingly, for a charged filament we have

$$\varphi(x, z = -\Delta) = -2\xi \mathrm{e}^{\kappa\Delta}\, \mathrm{Re}\left\{\mathrm{e}^{-\mathrm{i}\kappa x}\, \mathrm{Ei}[\kappa(x - \mathrm{i}\Delta)]\right\} \tag{6}$$

where $\mathrm{Ei}(t)$ is the exponential integral. After performing a summation over all the filaments arranged in the periodic set with period a, we arrive eventually at the formula for the SL potential and surface charge density of 2D electrons:

$$V(x) = \frac{\pi\xi}{\kappa a}\int_0^\infty \mathrm{d}z\, \frac{\mathrm{e}^{-z}\sinh[2\pi(\Delta + z/\kappa)/a]}{\cosh[2\pi(\Delta + z/\kappa)/a] - \cos^2(\pi x/a)} \tag{7}$$

$$\sigma(x) = \frac{\kappa}{2\pi} V(x). \tag{8}$$

At $\kappa \to \infty$, we immediately come back to the classical-limit equation (2). The quantum corrections are given by the κ-dependent terms in equation (7). The Fourier components $V^{(r)}$ of the potential (7) can be calculated analytically. For $V^{(r)}$ we find

$$V^{(r)} = \frac{2\pi\xi}{\kappa a}\, \frac{\exp(-2\pi\Delta|r|/a)}{1 + 2\pi|r|/\kappa a}. \tag{9}$$

We see from equation (9) that for a short-period SL, the quantum corrections may be rather important: the parameter $2\pi/\kappa a = \pi a_0^*/a$ for GaAs becomes unity for a period $a \sim 300$ Å. The depth of modulation of the lateral potential depends on the spacer thickness Δ exponentially; a rather simple estimate can be made in the nearly classical limit $\kappa a \gg 1$:

$$\frac{V_{\mathrm{min}}}{V_{\mathrm{max}}} = \tanh^2\left(\frac{\pi\Delta}{a}\right). \tag{10}$$

3. Dynamic conductivity of the 1D SL

In this section, expressions for components of the dynamic magnetoconductivity tensor $\hat{\sigma}(\omega, \boldsymbol{B})$ of a 1D lateral superlattice are derived†. This tensor describes the response of a system placed in an external magnetic field $\boldsymbol{B}$ to an alternating electric field of frequency ω. Our derivation generalizes the approach developed in [7] to the case of an external alternating electric field and an arbitrary periodic potential. It is based on a solution of the classical Boltzmann kinetic equation for 2D electrons subject to a laterally modulating potential $V(x)$, a constant uniform magnetic field $\boldsymbol{B}$ (directed along the z-axis) and a microwave field $\boldsymbol{E}(t) = \mathrm{Re}(\boldsymbol{E}_\omega \mathrm{e}^{-\mathrm{i}\omega t})$ ($\boldsymbol{E}_\omega$ is the complex amplitude of the electromagnetic field).

The linearized kinetic equation for a degenerate ($T = 0$ K) electron system reads

$$\begin{aligned}\hat{\mathcal{L}}_\omega \chi \equiv \left[v(x)\cos\varphi\,\frac{\partial}{\partial x} + \left(\frac{\partial v}{\partial x}\sin\varphi + \omega_{\mathrm{c}}\right)\frac{\partial\chi}{\partial\varphi} + \frac{1}{\tau}\left(1 - \int_0^{2\pi}\frac{\mathrm{d}\varphi}{2\pi}\right) - \mathrm{i}\omega\right]\chi_\omega \\ = -e\boldsymbol{E}_\omega \boldsymbol{u} v(x).\end{aligned} \tag{11}$$

Here $\chi(x, \varphi)$ is a nonequilibrium correction linear in $\boldsymbol{E}_\omega$ to the distribution function at the point x with the direction of the velocity (momentum) $\boldsymbol{u}$ ($\boldsymbol{u} = (\cos\varphi, \sin\varphi)$), $v(x) = v_{\mathrm{F}}\sqrt{1 + eV(x)/E_{\mathrm{F}}}$ is the magnitude of the electron velocity (v_{F} is the Fermi velocity), $\omega_{\mathrm{c}} = eB/m^*c$ is the cyclotron frequency and τ is the relaxation time (as in [7], τ is assumed to be constant). The complex amplitude of the current is expressed in terms of χ as

$$\boldsymbol{j}_\omega = -eN\langle\chi \boldsymbol{u} v\rangle \tag{12}$$

† Strictly speaking, the lateral potential depends on the magnetic field through the B-dependence of the screening (see, e.g., [12]). However, this dependence becomes important only for sufficiently strong fields when the density of states differs essentially from that of free-electron gas. The effects that we consider here occur in the regime of much weaker fields.

where $N = m/\pi\hbar^2$ is the density of states of an unmodulated 2D electron gas. The angular brackets mean averaging over x and φ:

$$\langle\cdots\rangle = \int_0^{2\pi} \frac{\mathrm{d}\varphi}{2\pi} \int \frac{\mathrm{d}x}{a} (\cdots). \tag{13}$$

Now we introduce (instead of χ_ω) a new unknown function $\boldsymbol{F}_\omega$ as follows:

$$\chi_\omega = -e \sum_{i,j} \left[\frac{2v(x)u_i}{v_{\mathrm{F}}^2} + F_{\omega i} \right] \frac{\sigma_{i,j}^{(0)}(\omega, \boldsymbol{B})}{Ne^2} E_{\omega j} \tag{14}$$

where $\sigma_{i,j}^{(0)}(\omega, \boldsymbol{B}) = \sigma_0 \eta d_{i,j}/(\eta^2 + \gamma^2)$ is the unperturbed (i.e. at $V(x) = 0$) dynamic (Drude-type) magnetoconductivity tensor of the 2D system, $\sigma_0 = N_{\mathrm{s}} e^2 \tau / m^*$ is the static conductivity of the 2D electron gas, $N_{\mathrm{s}} = N E_{\mathrm{F}}$ is the surface electron density, $d_{xx} = d_{yy} = 1$, $d_{yx} = -d_{xy} = \gamma/\eta$, $\gamma = \omega_{\mathrm{c}}\tau$ and $\eta = 1 - \mathrm{i}\omega\tau$.

The first term in equation (14) gives $\sigma_{i,j}^{(0)}(\omega, \boldsymbol{B})$. The second leads to a correction associated with $V(x)$:

$$\Delta\sigma_{i,j}(\omega, \boldsymbol{B}) = \sum_k \langle u_i v F_{\omega k} \rangle \sigma_{k,j}^{(0)}. \tag{15}$$

It is evident that $F_{\omega i} = F_\omega \delta_{xi}$ for a 1D lateral SL. Substituting (14) into (11) we obtain the following equation for F_ω:

$$\hat{\mathcal{L}}_\omega F_\omega = -\frac{e}{E_{\mathrm{F}}} \frac{\partial V(x)}{\partial x}. \tag{16}$$

Using equation (11), the expression for $\Delta\sigma_{i,j}$ can be readily transformed into

$$\Delta\sigma_{i,j} = \frac{1}{N_{\mathrm{s}} e} \sigma_{ix}^{(0)} \sigma_{xj}^{(0)} \left\langle F \frac{\partial V}{\partial x} \right\rangle. \tag{17}$$

We assume the lateral potential $V(x)$ to be weak. This allows solving equation (16) perturbatively, and, to first order in V, the function F is determined by

$$\hat{\mathcal{L}}_\omega^{(0)} F_\omega = -\frac{e}{E_{\mathrm{F}}} \frac{\partial V(x)}{\partial x} \tag{18}$$

with $\hat{\mathcal{L}}_\omega^{(0)} = \hat{\mathcal{L}}_\omega \big|_{V=0}$. From equation (18) we can find the following equation for spatial Fourier components of the function $F(x, \varphi)$ ($F = \sum_{r=-\infty}^{\infty} F^{(r)} \exp(\mathrm{i}rqx)$, $q = 2\pi/a$):

$$\mathrm{i}rql \cos\varphi F^{(r)} + \omega_{\mathrm{c}} \frac{\partial F^{(r)}}{\partial \varphi} + \eta F^{(r)} - \overline{F^{(r)}} = -\frac{\mathrm{i}rqe\tau V^{(r)}}{E_{\mathrm{F}}} \tag{19}$$

where the operation $\overline{(\cdots)}$ means averaging over φ, and $l = v_{\mathrm{F}}\tau$ is the free path length. Solving equation (19), we have

$$\overline{F^{(r)}} = -\frac{\mathrm{i}rqleV^{(r)}}{v_{\mathrm{F}} E_{\mathrm{F}}} \frac{S(rqv_{\mathrm{F}}/\omega_{\mathrm{c}})}{1 - S(rqv_{\mathrm{F}}/\omega_{\mathrm{c}})} \tag{20}$$

$$S(z) = \eta \sum_{n=-\infty}^{\infty} \frac{J_n^2(z)}{n^2\gamma^2 + \eta^2}. \tag{21}$$

Here $J_n(z)$ are Bessel functions. Now we substitute (20) into (17) and come finally to

$$\sigma_{i,j}(\omega, \boldsymbol{B}) = \frac{\sigma_0 \eta}{\eta^2 + \gamma^2} \left\{ d_{i,j} - \frac{\eta d_{ix} d_{xj} q^2 l^2}{2(\eta^2 + \gamma^2)} \sum_{r=-\infty}^{\infty} r^2 \frac{|eV^{(r)}|^2}{E_{\mathrm{F}}^2} \frac{S(rqv_{\mathrm{F}}/\omega_{\mathrm{c}})}{1 - S(rqv_{\mathrm{F}}/\omega_{\mathrm{c}})} \right\}. \tag{22}$$

Using equation (22), we can easily verify that at $\omega = 0$ the expression for $\rho_{i,j}$ ($\hat{\rho} = \hat{\sigma}^{-1}(0, B)$ is the static magnetoresistivity) reduces to the result obtained in [7]. At $B = 0$, equation (22) can be somewhat simplified:

$$\sigma_{i,j}(\omega) = \sigma_0 \left\{ \frac{1}{\eta}\delta_{i,j} - \frac{q^2 l^2}{2\eta^2} \sum_{r=-\infty}^{\infty} r^2 \frac{|eV^{(r)}|^2}{E_{\mathrm{F}}^2} \frac{C(rql)}{1 - C(rql)} \delta_{ix}\delta_{jx} \right\} \tag{23}$$

where

$$C(t) = 1/\sqrt{\eta^2 + t^2}.$$

4. Magnetoreflectance and the Faraday effect

Let us consider the manifestation of the lateral modulation of the 2D electron gas in reflection from the system under discussion. We solve the electrodynamic problem in which the 2D electron system with a spacer is modelled by a δ-like layer on a semi-infinite dielectric substrate with refractive index n. The fields are subject to corresponding boundary conditions and we can easily arrive at formulae for the reflectance R, Faraday rotation angle ϕ and ellipticity δ in terms of the components of the dynamic conductivity tensor. For normal incidence of the wave polarized along the x-axis we have

$$R_{\|} = \frac{|(n + 1 + \Sigma_{yy})(1 - n + \Sigma_{xx}) - \Sigma_{yx}^2|^2 + 4|\Sigma_{yx}|^2}{|(n + 1 + \Sigma_{xx})(n + 1 + \Sigma_{yy}) + \Sigma_{yx}^2|^2} \tag{24}$$

$$|\phi_{\|}| = 2 \left| \mathrm{Re}\left(\frac{\Sigma_{yx}}{(n + 1 + \Sigma_{yy})(1 - n - \Sigma_{xx}) - \Sigma_{yx}^2} \right) \right| \tag{25}$$

$$\delta_{\|} = 2\,\mathrm{Im}\left(\frac{\Sigma_{yx}}{(n + 1 + \Sigma_{yy})(1 - n - \Sigma_{xx}) - \Sigma_{yx}^2} \right). \tag{26}$$

Here we introduced the notation $\Sigma_{i,j} = 4\pi\sigma_{i,j}/c$. In the case of an incident wave polarized along the y-axis, the expressions for $R_{\perp}$ and $\phi_{\perp}$ can be obtained from (24) and (24) by making the interchange $x \leftrightarrow y$.

The results of numerical calculations of R and ϕ are shown in figures 2–5. The calculations were performed using the following parameters:

$$\begin{aligned}
&\mu = 5 \times 10^4\ \mathrm{cm^2\ V^{-1}\ s^{-1}}\ (\tau = 1.9\ \mathrm{ps}) \\
&m^* = 0.067 m_0 \\
&N_{\mathrm{s}} = 4 \times 10^{11}\ \mathrm{cm^{-2}}\ (E_{\mathrm{F}} = 14.29\ \mathrm{meV},\ v_{\mathrm{F}} = 2.74 \times 10^7\ \mathrm{cm\ s^{-1}}) \\
&a_0^* = 10.12\ \mathrm{nm} \\
&a = 32\ \mathrm{nm} \\
&\Delta = 75\ \text{Å} \\
&\xi = 2 \times 10^5\ \text{electrons cm}^{-1} \\
&n = 3.58.
\end{aligned}$$

$\Delta R_{\|}$ and $\Delta R_{\perp}$ ($\Delta R = R - R^{(0)}$ is the lateral correction to the reflectance $R^{(0)}$ of an unmodulated electron gas) are plotted at $B = 0$ as functions of frequency ω in figure 2. For a y-polarized incident wave we have the conventional monotonic Drude behaviour $R_{\perp} \equiv R^{(0)}(\omega)$ ($\Delta R_{\perp}$ is negligibly small), but for the x-polarized case the effect of the lateral SL results in a $\Delta R_{\|}(\omega)$ curve with a maximum at $\omega\tau \sim 1$.

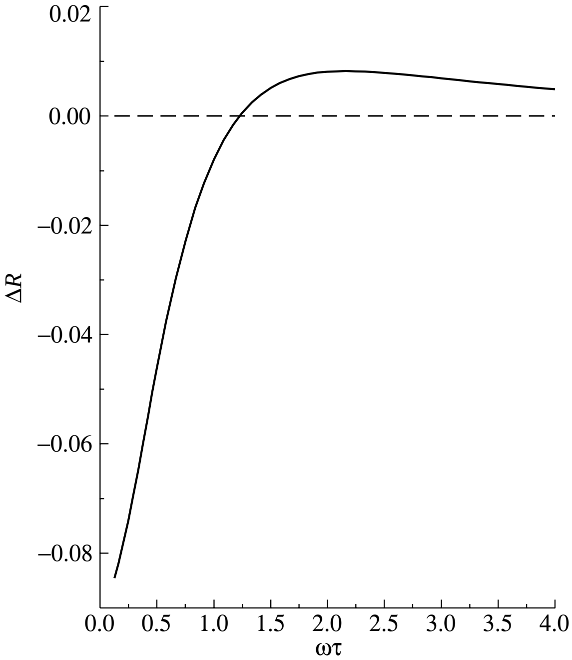

Figure 2. The frequency dependence of the reflectance at $B = 0$. In this and the following figures, full and broken curves correspond to $\Delta R_{\parallel}$ ($\Delta R_{\perp}$) and $\Delta R = R - R^{(0)}$, respectively.

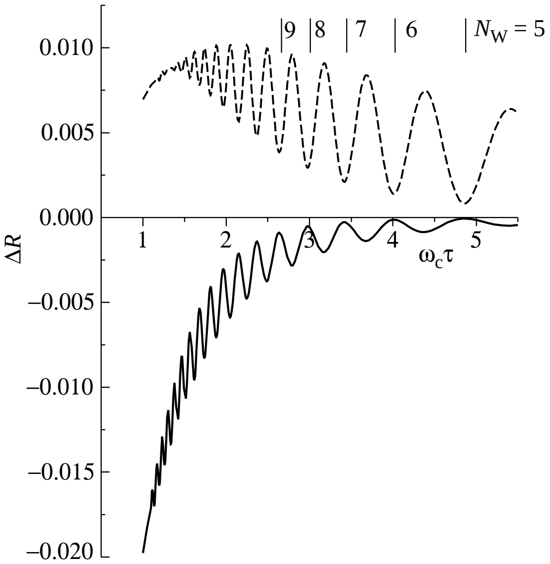

Figure 3. The magnetic field dependence of $\Delta R(B)$ for $\omega\tau = 0.4$. Ticks indicate extremum positions of Weiss oscillations in accordance with the relation $2l/a = (N_W - 1/4)\gamma$, $N_W = 1, 2, 3, \ldots$.

Figures 3–5 demonstrate the magnetic field dependence of the reflection coefficients for three different frequencies. The curves in figure 3 ($\omega\tau \ll 1$) show oscillatory behaviour. The

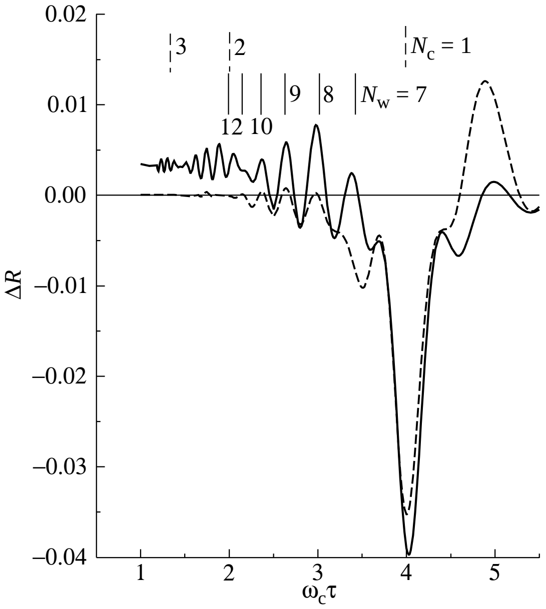

Figure 4. The magnetoreflectance (as in figure 3) for $\omega\tau = 4$. Full ticks indicate the positions of Weiss oscillations; broken ticks correspond to CR harmonics ($\omega_c N_c = \omega$, $N_c = 1, 2, \ldots$).

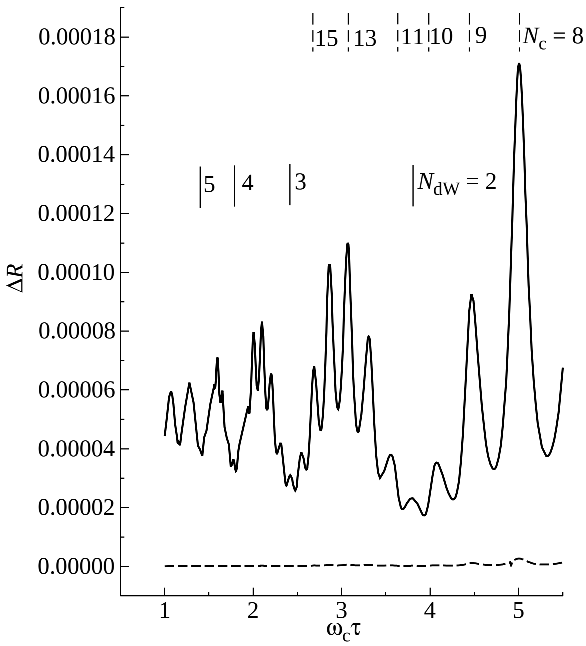

Figure 5. Reflectance versus B for $\omega\tau = 40$. Full ticks indicate the positions of dynamic Weiss oscillations following from (27); broken ticks are as in figure 4.

positions of $R_{\parallel}(B)$ maxima (and, correspondingly, $R_{\perp}(B)$) minima) coincide with the positions of static-magnetoresistivity $\rho_{xx}(B)$ minima in Weiss oscillations. At high frequencies,

$1 \ll \omega\tau \ll \omega_{cr}\tau$, where $\omega_{cr} = 2\pi v_F/a$ (for the chosen parameters, $\omega_{cr}\tau = 72.4$), we have beats (figure 4): the envelope function associated with the cyclotron resonance (CR) and its harmonics modulates the Weiss oscillations.

Figure 5 plots $R(B)$ for $\omega\tau = 40$ (i.e., at $\omega \lesssim \omega_c$). At the given frequency the CR harmonics are modulated by the envelope function with minima obeying the relation

$$2\frac{v_F}{a}\Phi\left(\frac{\omega}{\omega_{cr}}\right) = \omega_c\left(N_{dW} - \frac{1}{4}\right) \tag{27}$$

where $\Phi(x) = \sqrt{1-x^2} - x \arctan(1/x^2 - 1)$, $N_{dW} = 1, 2, \ldots$. Here we have a manifestation of the so-called dynamic Weiss oscillations. The possibility of observing of Weiss-type oscillations in a dynamic regime was predicted in [9]. At $\omega > \omega_{cr}$, only CR harmonics with an exponential envelope function are left (no Weiss oscillations occur).

Similar oscillatory behaviour due to the periodic lateral potential can also be observed in other magneto-optical quantities, such as the transmittance and Faraday rotation angle. As an example, figure 6 shows the B-dependence of the Faraday rotation angle and ellipticity in a reflected electromagnetic wave. On the scale adopted, the results for the two polarizations coincide very nicely.

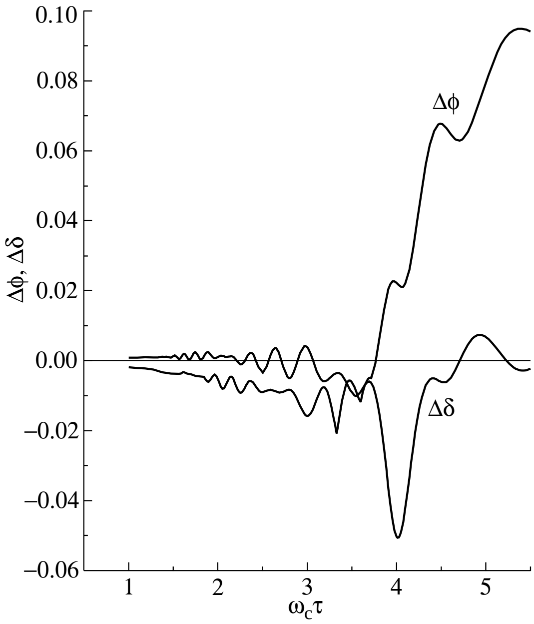

Figure 6. The magnetic field dependence of the Faraday rotation angle $\Delta\phi$ and ellipticity $\Delta\delta$ for $\omega\tau = 4$.

In conclusion, we report an exact analytical solution of the Thomas–Fermi electrostatic problem for a 2D electron gas in a 1D lateral superlattice. The manifestations of the SL potential in the magnetoreflectance and Faraday effect are considered. In both cases the oscillatory behaviour of the observable values is established.

Acknowledgments

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